# Chapter 5
# AM, PM, and digital modulated systems

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- Introduction
- Amplitude modulation
- Double-sideband Suppressed Carrier
- Asymmetric sideband signals
- Vestigial sideband signals

# Introduction

- **modulated signal**

$$s(t) = \mathrm{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

- **complex envelope $g(t)$ is a function of the modulating signal $m(t)$:**

$$g(t) = g\left[ m(t) \right]$$

**$g[.]$ performs a mapping operation on $m(t)$**

- **Modulation is the process of imparting the source information onto a Band-pass signal with a carrier frequency $f_c$ by the introduction of amplitude or phase perturbations or both.**

# Classification of modulation

## according to modulating signal $m(t)$ :

- Analog modulation
- Digital modulation
  - Binary modulation
  - Multilevel modulation

## according to carrrier $A_c \cos(\omega_c t + \theta_0)$ :

- Amplitude modulation (AM)
- Phase modulation (PM)
- Frequency modulation (FM)

# Introduction

*The goals of this chapter are to*

- Study g(t) and s(t) for various types of analog and digital modulations
- evaluate the spectrum for various types of analog and digital modulations
- examine some transmitter and receiver structures

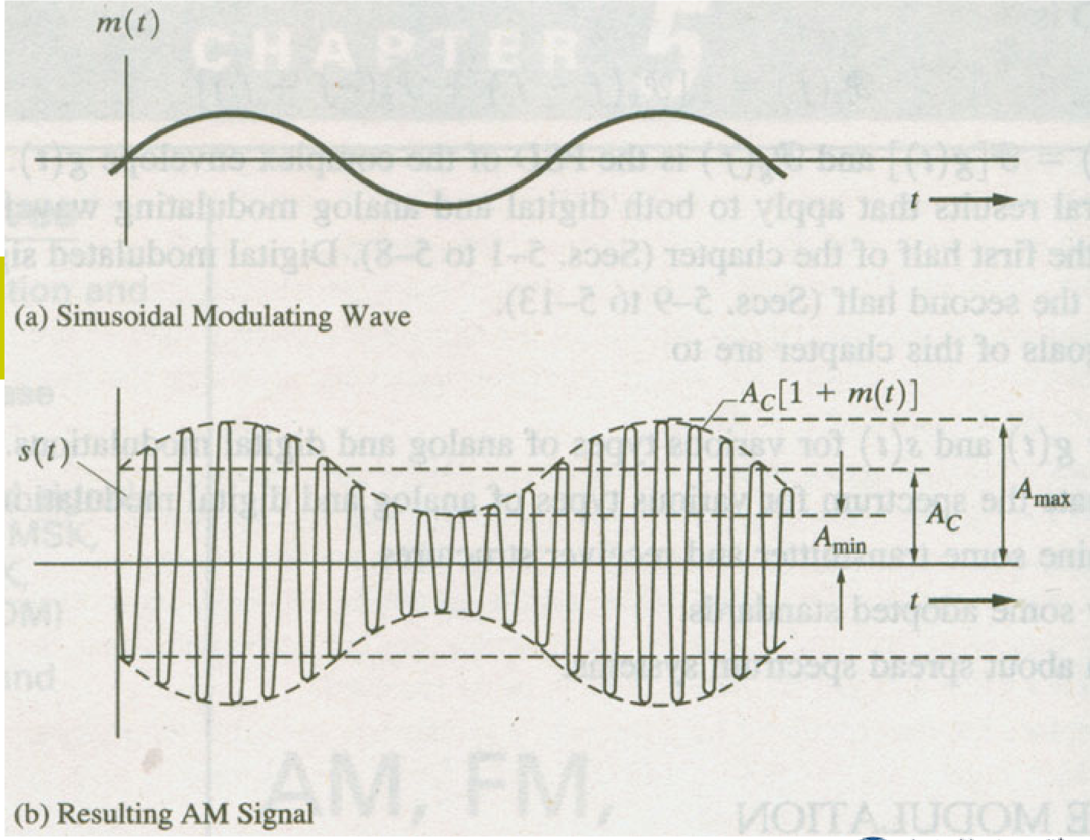
# 5.1 Amplitude modulation （AM）

## The complex envelope of an AM signal:

$$g(t) = A_c[1 + m(t)]$$

**AM signal:**

$$s(t) = A_c[1 + m(t)]\cos\omega_c t$$



# Amplitude modulation

## Definition:

$$\%\text{positive modulation} = \frac{A_{max} - A_c}{A_c} \times 100 = \max[m(t)] \times 100$$

$$\%\text{negative modulation} = \frac{A_c - A_{min}}{A_c} \times 100 = -\min[m(t)] \times 100$$

$$\%\text{modulation} = \frac{A_{max} - A_{min}}{2A_c} \times 100 = \frac{\max[m(t)] - \min[m(t)]}{2} \times 100$$

$A_{\max}$ : $\max\{ A_c[1+m(t)] \}$ $A_{\min}$ : $\min\{ A_c[1+m(t)] \}$

$A_c$ : $A_c[1+m(t)],\ \ when\ \ m(t)=0$

# Amplitude modulation

*Example*: **5-5**

**A 50000w AM broadcast transmitter is being evaluated by means of a two-tone test. The transmitter $m(t) = A_1 \cos \omega_1 t + A_1 \cos 2\omega_1 t$ oad, and where $f_1$=500Hz. Assume that a pefect AM signal is generated.**

**Determine:**

**(a)Evaluate the complex envelope for the AM signal in terms of $A_1$ and $\omega_1$.**

**(b)Determine the value of $A_1$ for 90% modulation.**

**(c) find the value of the peak current and average current into the 50 Ω load for the 90% modulation case.**

## Detection

- If the percentage of negative modulation is less than 100%, an envelope detector may be used to recover the modulation without distortion;

- If the percentage of negative modulation is over 100%, undistorted modulation can still be recovered provided the product detector is used.

- A product detector is superior to an envelope detector when the input signal-to-noise ratio is small.

# Amplitude modulation

- **normalized average power**

$$\langle s^2(t)\rangle = \frac{1}{2}\langle |g(t)|^2\rangle = \frac{1}{2}A_c^2\langle [1+m(t)]^2\rangle$$

$$= \frac{1}{2}A_c^2\langle 1+2m(t)+m^2(t)\rangle$$

$$= \frac{1}{2}A_c^2 + A_c^2\langle m(t)\rangle + \frac{1}{2}A_c^2\langle m^2(t)\rangle$$

- **If the modulation contains no dc level, then ⟨m(t)⟩ = 0, and the normalized power of the AM signal is**

$$\langle s^2(t)\rangle = \underbrace{1/2A_c^2}_{discrete\ \text{carrier power}} \underbrace{+1/2A_c^2\langle m^2(t)\rangle}_{sideband\ power}$$

# Amplitude modulation

- **modulation efficiency**

$$E = \frac{\left\langle m^2(t) \right\rangle}{1 + \left\langle m^2(t) \right\rangle} \times 100\%$$

- **The highest efficiency that can be attained for a 100% AM signal would be 50%.**

- **The normalized peak envelope power (PEP) of the AM signal:**

$$P_{PEP} = \frac{A_c^2}{2}\{1 + \max[m(t)]\}^2$$

# Amplitude modulation

- **Spectrum**

$$g(t) = A_c[1 + m(t)]$$

$$G(f) = A_c[\delta(f) + M(f)]$$

$$S(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

$$\delta(-f) = \delta(f)$$

$$M^*(-f) = M(f)$$

$$S(f) = \frac{Ac}{2}[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)]$$

# Amplitude modulation

- **Spectrum**

$$S(f)=\frac{A_c}{2}[\delta(f-f_c)+M(f-f_c)+\delta(f+f_c)+M(f+f_c)]$$

- **The AM spectrum is just a translated version of the modulation spectrum plus delta functions that give the carrier line spectral component.**
- **The bandwidth is twice that of the modulation**

# Amplitude modulation

- **Spectrum**

***Example***

**Evaluate the magnitude spectrum for an amplitude-modulated (AM) signal.**

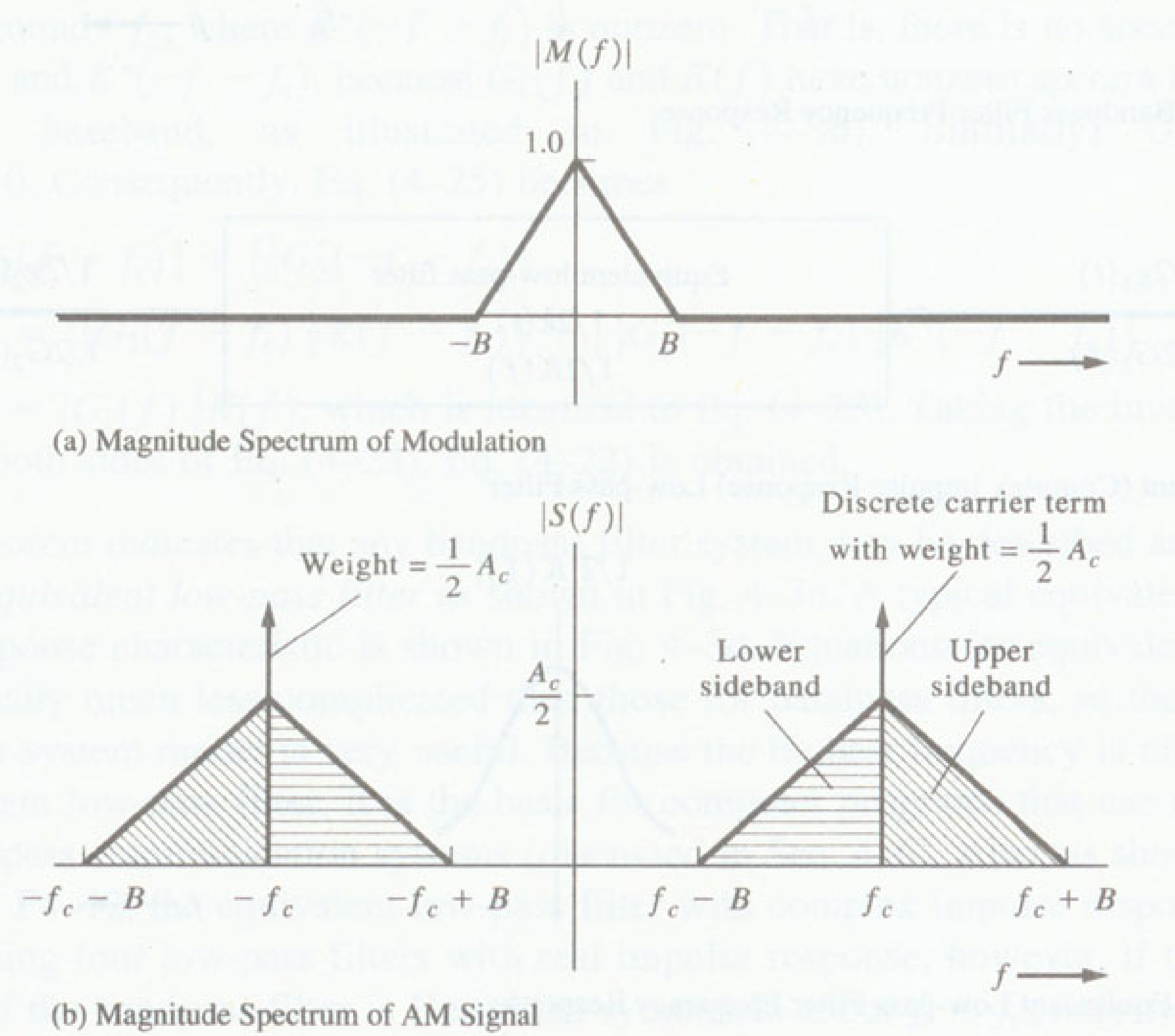


**Figure 4–2** Spectrum of AM signal.

# Amplitude modulation

## *Example*: power of an AM signal

**Problem:** suppose that a **5000w AM transmitter** connected to a **50Ω load**, $A_c$ is given by $\frac{1}{2}A_c^2/50=5000, A_c=707(V)$, If the transmitter is **100% modulated** by a 1000 Hz test tone.

**Determine:**

- The **total average power**.
- The **peak envelope power** (PEP) on the 50 Ω load.
- The **modulation efficiency**.

# 5.3 Double-sideband Suppressed Carrier（DSB-SC）

# Double-sideband Suppressed Carrier

A double-sideband suppressed carrier (DSB-SC) signal is an AM signal that has a suppressed discrete carrier.

The complex envelope : $g(t) = A_c m(t)$

DSB-SC signal: $s(t) = A_c m(t) \cos \omega_c t$

- The voltage spectrum of the DSB-SC signal is

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

# Double-sideband Suppressed Carrier

## *Characteristic*

- The modulation efficiency of a DSB-SC is 100%
- A product detector (which is more expensive than an Envelope detector) is required for demodulation of the DSB-SC signal
- The sideband power of a DSB-SC signal is 4 times that of a comparable AM signal with the some peak level.

# 5.5 Asymmetric sideband signals （SSB）

# Asymmetric sideband signals

## Signal sideband

### *definition*

- An upper single sideband (USSB) signal has a zero-valued spectrum for $|f| < f_c$, where $f_c$ is the carrier frequency.

- A lower signal sideband (LSSB) signal has a zero-valued spectrum for $|f| > f_c$, where $f_c$ is the carrier frequency.

# Asymmetric sideband signals

## Signal sideband

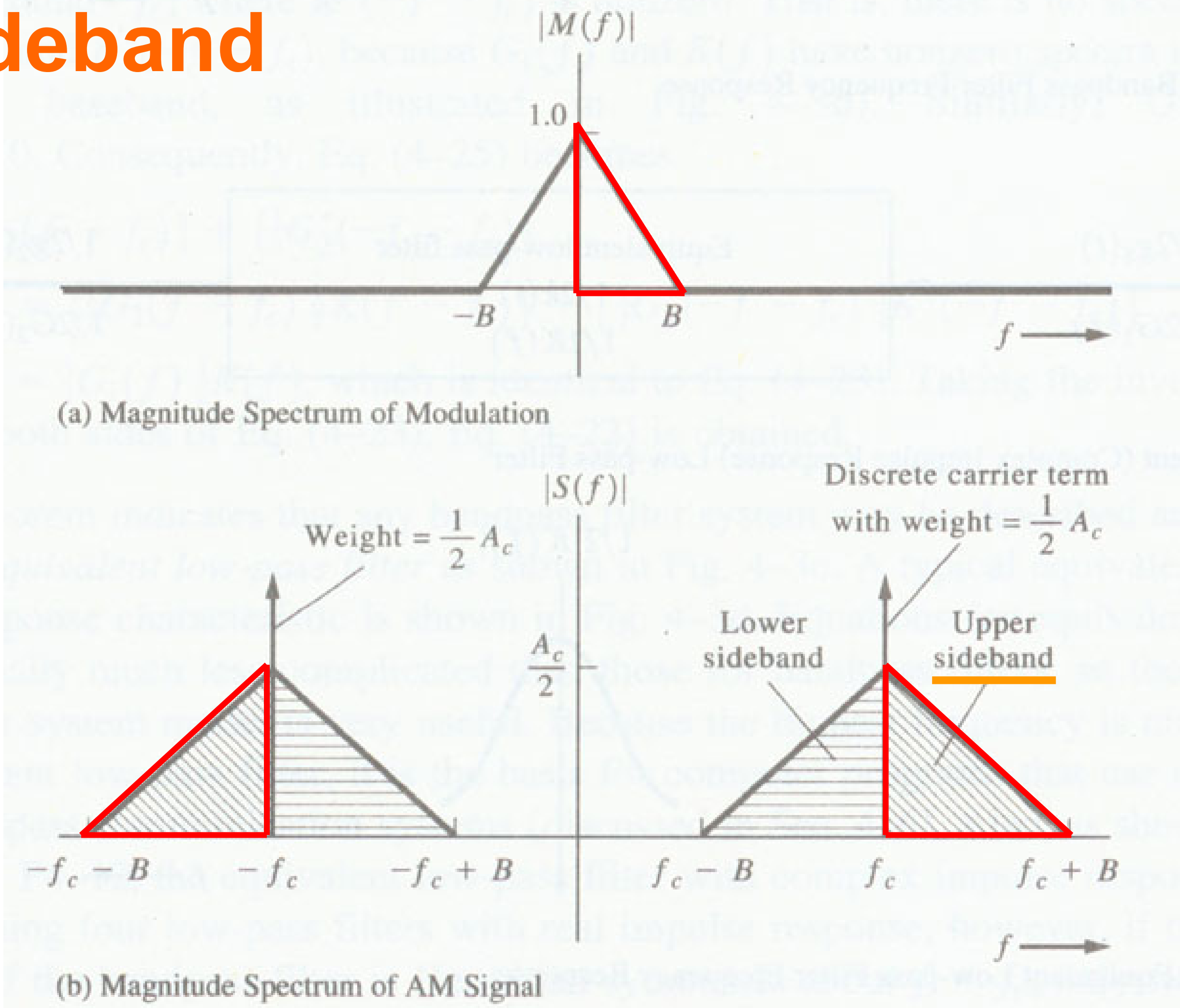



**Figure 4–2** Spectrum of AM signal.

# Asymmetric sideband signals

The **complex envelope of an SSB** signal is given by

$$g(t) = A_c[m(t) \pm j\hat{m}(t)]$$

Which results in the **SSB signal waveform**:

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$

Where $\hat{m}(t)$ Denotes the **Hilbert transform** of ***m*( *t* )**, which is given by:

$$\hat{m}(t) = m(t) * h(t)$$

Where ***h*(*t*)=1/( π *t*)**, and ***H*( *f* ) = ℱ [ *h*(*t*) ]** corresponds to a -90$^{0}$ phase-shift network:

$$H(f) = \begin{cases} -j & f > 0 \\ j & f < 0 \end{cases}$$

# Asymmetric sideband signals

## *Proof for USSB signal*

$$g(t) = A_c[m(t) + j\hat{m}(t)]$$

$$\hat{m}(t) = m(t) * h(t)$$

$$G(f) = A_c[M(t) + jF[\hat{m}(t)]]$$

$$F[\hat{m}(t)] = M(f)H(f)$$

$$G(f) = A_c[M(f) + jM(f)H(f)] = A_c M(f)[1 + jH(f)]$$

$$H(f) = \begin{cases} -j & f > 0 \\ j & f < 0 \end{cases}$$

$$G(f) = \begin{cases} 2A_c M(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

$$G^*(f) = A_c[M^*(f) - jM^*(f)H^*(f)]$$

$$G^*(f) = \begin{cases} 2A_c M^*(f) , & f > 0 \\ 0 , & f < 0 \end{cases}$$

$$S(f) = \frac{1}{2}G(f - f_c) + \frac{1}{2}G^*(-f - f_c)$$

$$S(f) = A_c\begin{Bmatrix} M(f - f_c), & f > f_c \\ 0 , & f < f_c \end{Bmatrix} + A_c\begin{Bmatrix} 0 , & f > -f_c \\ M(f + f_c) , & f < -f_c \end{Bmatrix}$$

# Asymmetric sideband signals

- The **normalized average power** of the SSB signal

$$\langle s^2(t)\rangle = \frac{1}{2}\langle |g(t)|^2\rangle = \frac{1}{2}A_c^2\langle |m(t)+j\hat{m}(t)|^2\rangle$$

$$= \frac{1}{2}A_c^2\langle m^2(t)+\hat{m}^2(t)\rangle$$

$$= A_c^2\langle m^2(t)\rangle$$

- The **normalized peak envelope power** (PEP) is :

$$PEP = \frac{1}{2}\max\{|g(t)|^2\} = \frac{1}{2}A_c^2\max\{m^2(t)+[\hat{m}(t)]^2\}$$

# Asymmetric sideband signals

## Characteristic of the SSB signal

- **SSB signal have both AM and PM**

**For the AM component (real envelope):**

$$R(t) = |g(t)| = A_c^2 \sqrt{m^2(t) + [\hat{m}(t)]^2}$$

**For the PM component:**

$$\theta(t) = \angle g(t) = \tan^{-1}\left[\frac{\pm \hat{m}(t)}{m(t)}\right]$$

# Asymmetric sideband signals

## Characteristic of the SSB signal

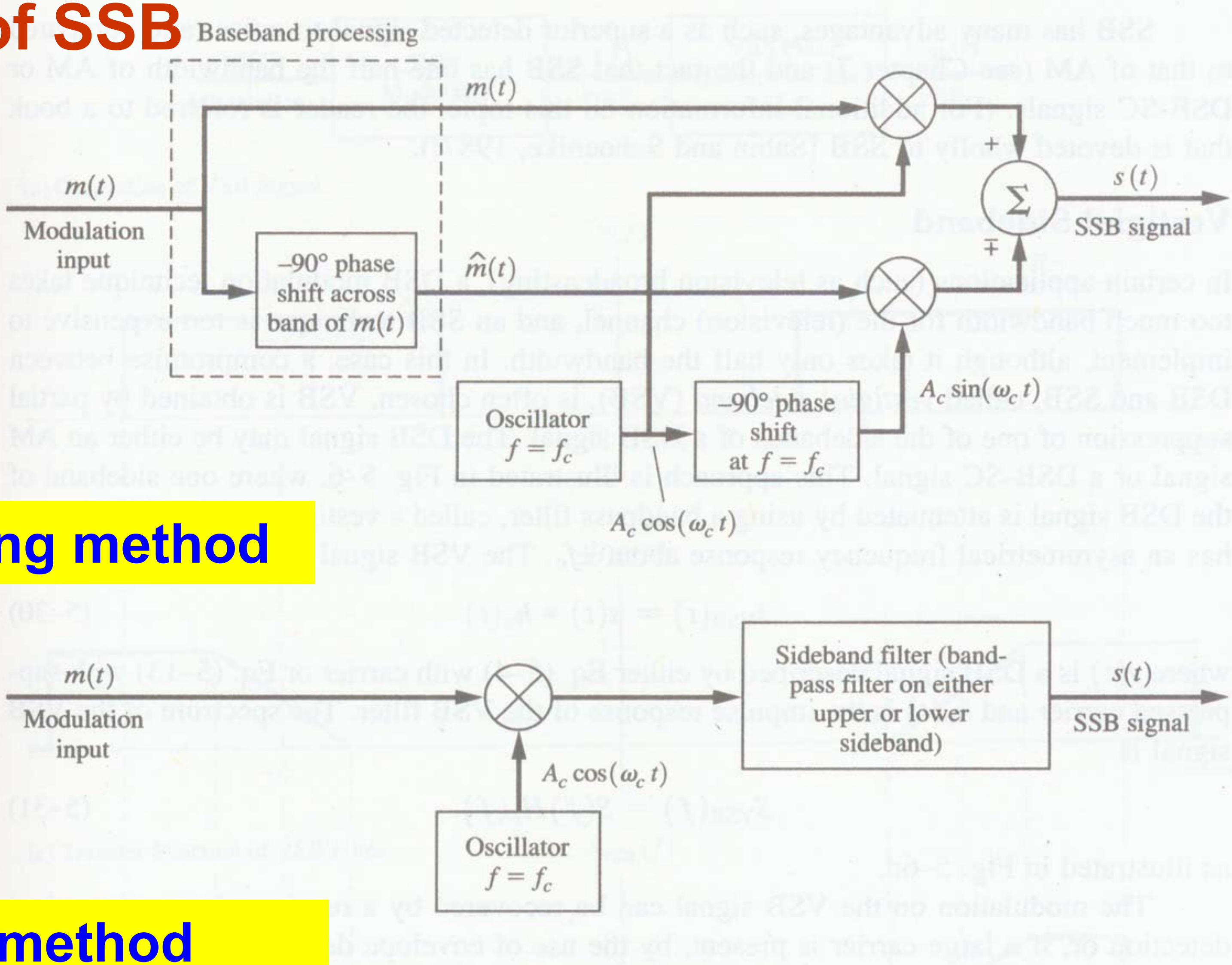
- superior detected signal-to-noise compared to that of AM

- The bandwidth is the same as that of the modulating signal (which is half the bandwidth of an AM or DSB-SC signal).

- The SSB signal can be demodulated by using a coherent detector, not a simple envelope detector.

# Asymmetric sideband signals

## Generation of SSB

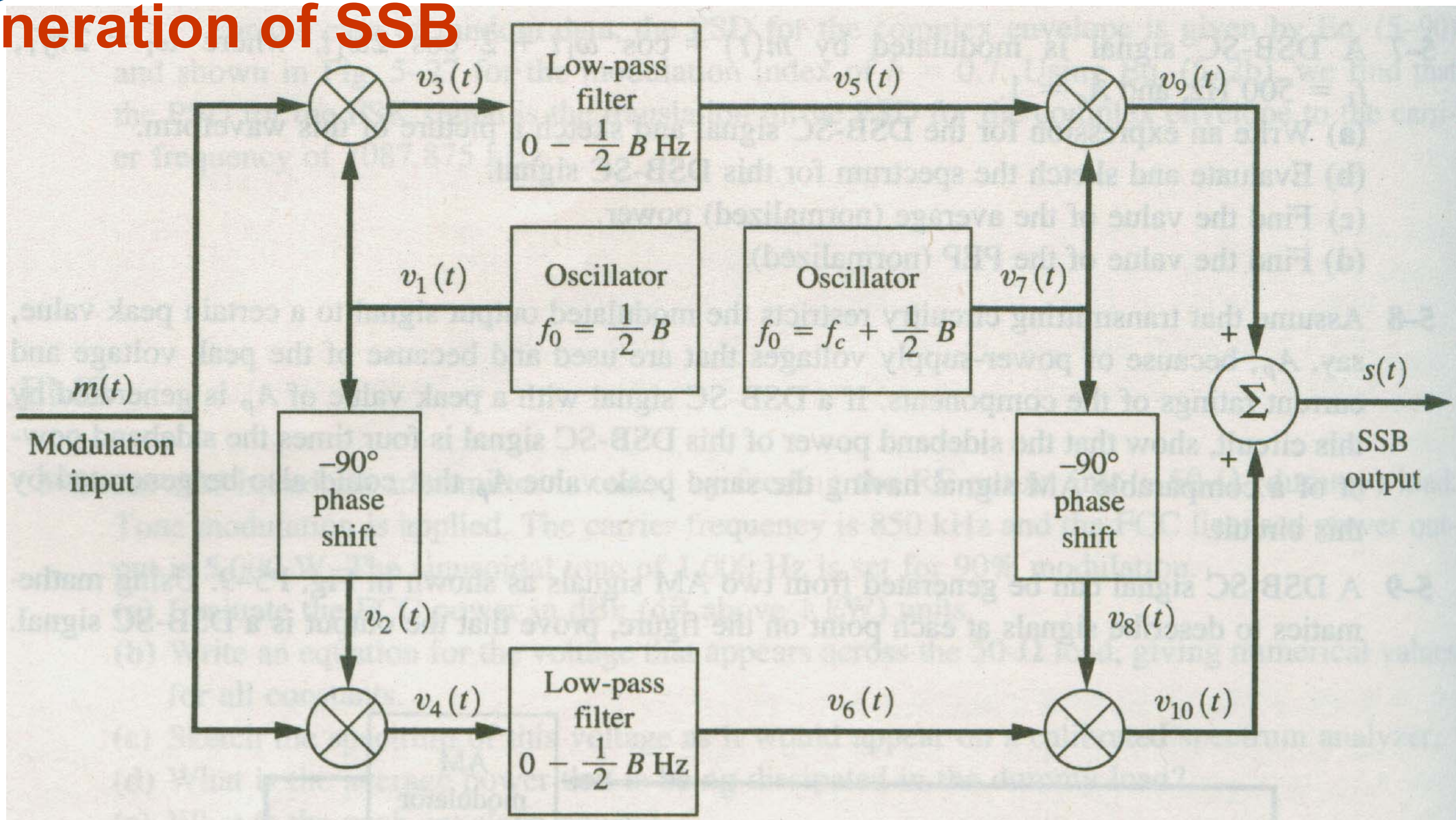



**a. Phasing method**

**b. Filter method**

**Figure 5–5** Generation of SSB.

## Generation of SSB




**c. Weaver's method**

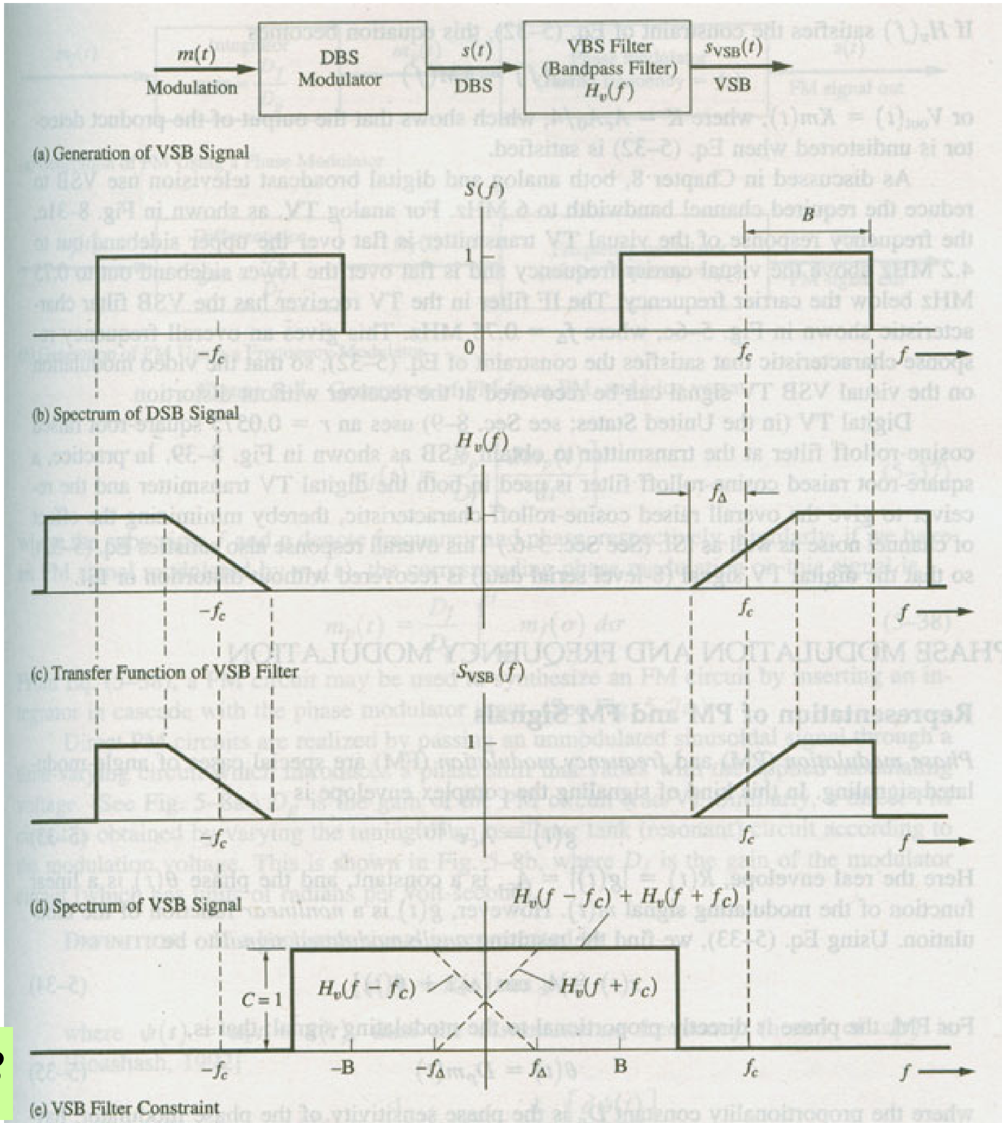
# Vestigial sideband signals

**Vestigial sideband (VSB)** is obtained by **partial suppression** of one of the sidebands of a DSB signal.

$$s_{VSB}(t) = s(t) * h_v(t)$$

For recovery of undistorted modulation, **VSB filter must satisfy the constraint:**

$$H_v(f - f_c) + H_v(f + f_c) = C, \qquad |f| \le B$$




(a) Generation of VSB Signal

(b) Spectrum of DSB Signal

(c) Transfer Function of VSB Filter

(d) Spectrum of VSB Signal

(e) VSB Filter Constraint

# Summary

**AM**

$$g(t) = A_c[1+m(t)]$$

$$s(t) = A_c[1+m(t)]\cos\omega_c t$$

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + M(f-f_c) + \delta(f+f_c) + M(f+f_c)]$$

$$p = \underbrace{1/2A_c^2}_{\text{discrete carrier power}} \underbrace{+1/2A_c^2\langle m^2(t)\rangle}_{\text{sideband power}}$$

**Bandwidth: 2B**

**DSB-SC**

$$g(t) = A_c m(t)$$

$$s(t) = A_c m(t)\cos\omega_c t$$

$$S(f) = 1/2A_c^2[M(f-f_c) + M(f+f_c)]$$

$$p = (1/2)A_c^2\langle m^2(t)\rangle$$

**Bandwidth: 2B**

**SSB-AM**

$$g(t) = A_c[m(t) \pm j\hat{m}(t)]$$

$$s(t) = A_c[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t]$$

$$S(f) = \begin{cases} A_c M(f-f_c) & f > f_c \\ 0 & f < f_c \end{cases} + \begin{cases} 0 & f > -f_c \\ A_c M(f+f_c) & f < -f_c \end{cases}$$

$$p = A_c^2\langle m^2(t)\rangle$$

**Bandwidth: B**

# 5.6 Phase Modulation and Frequency Modulation

## content

- Representation of PM and FM signal
- Spectrum of angle-modulation signals
- Frequency-Division multiplexing

# Representation of PM and FM signal

The complex envelope of the Angle-modulated signal is:

$$g(t) = A_c e^{j\theta(t)}$$

**Angle-modulated signal is:**

$$s(t) = A_c \cos[\omega_c t + \theta(t)]$$

**For PM, the phase is directly proportional to *m*(*t*)**

$$\theta(t) = D_p m(t)$$

**For FM, the phase is proportional to the integral of *m*(*t*)**

$$\theta(t) = D_f \int_{-\infty}^{t} m(\sigma) d\sigma$$

# PM & FM

## Relationship between PM and FM

Phase $\xrightarrow{d/dt}$ frequency

frequency $\xrightarrow{\int dt}$ Phase

**PM:** $\theta(t) = D_p m(t)$

**FM:** $\theta(t) = D_f \int_{-\infty}^{t} m(\sigma)d\sigma$

$$\theta_{PM}(t) = \theta_{FM}(t)$$

**PM by $m_p(t)$, there is also FM, the corresponding FM by $m_f(t)$ is:**

$$m_f = \frac{D_p}{D_f}\left[\frac{dm_p(t)}{dt}\right]$$

**FM by $m_f(t)$, there is also PM, the corresponding PM by $m_p(t)$ is :**

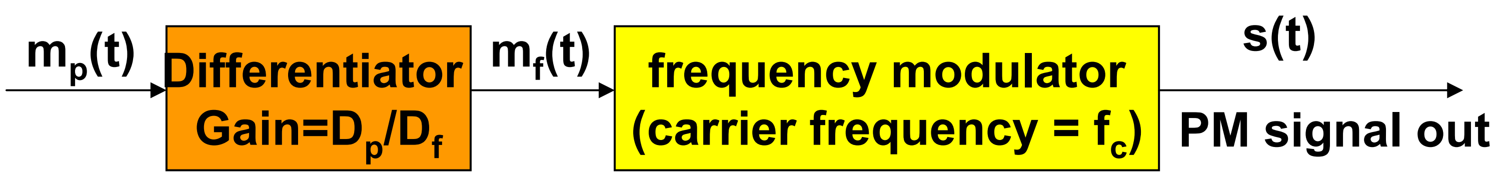
$$m_p = \frac{D_f}{D_p}\int_{-\infty}^{t} m_f(\sigma)d\sigma$$

# PM & FM

## Generation of FM from PM， and vice versa

### a) Generation of FM using a phase Modulator


### b) Generation of PM using a frequency Modulator



# PM & FM

## Definition:

If a band-pass signal is represented by :

$$s(t) = A_c \cos[\omega_c t + \theta(t)]$$

$$= R(t)\cos\psi(t) \qquad \text{where } \Psi(t) = \omega_c t + \theta(t)$$

then the instantaneous frequency (Hz) of $s(t)$ is:

$$f_i(t) = \frac{1}{2\pi}[\frac{d\psi(t)}{dt}] = f_c + \frac{1}{2\pi}[\frac{d\theta(t)}{dt}]$$

*Example:*

**For FM:**

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

$$f_i(t) = f_c + \frac{1}{2\pi} D_f m(t)$$

**The frequency deviation from the carrier frequency is:**

$$f_d = f_i(t) - f_c = \frac{1}{2\pi}[\frac{d\theta(t)}{dt}]$$

**The peak frequency deviation is :**

$$\Delta F = \max\{\frac{1}{2\pi}[\frac{d\theta(t)}{dt}]\}$$

**The peak-to-peak deviation is**

$$\Delta F_{pp} = \max\{\frac{1}{2\pi}[\frac{d\theta(t)}{dt}]\} - \min\{\frac{1}{2\pi}[\frac{d\theta(t)}{dt}]\}$$

# PM & FM

## *Example:*

**For the case of FM, the instantaneous frequency is:**

$$f_i(t) = f_c + \frac{1}{2\pi}[\frac{d\theta(t)}{dt}] = f_c + \frac{1}{2\pi} D_f m(t)$$

**$f_i(t)$ varies about the assigned carrier frequency $f_c$ in a manner that is directly proportional to the modulated signal $m(t)$.**

**The peak frequency deviation is :**

$$\Delta F = \frac{1}{2\pi} D_f V_p$$

**where** $V_p = \max[m(t)]$

# PM & FM

**An increase in the amplitude of the modulation signal $V_p$**

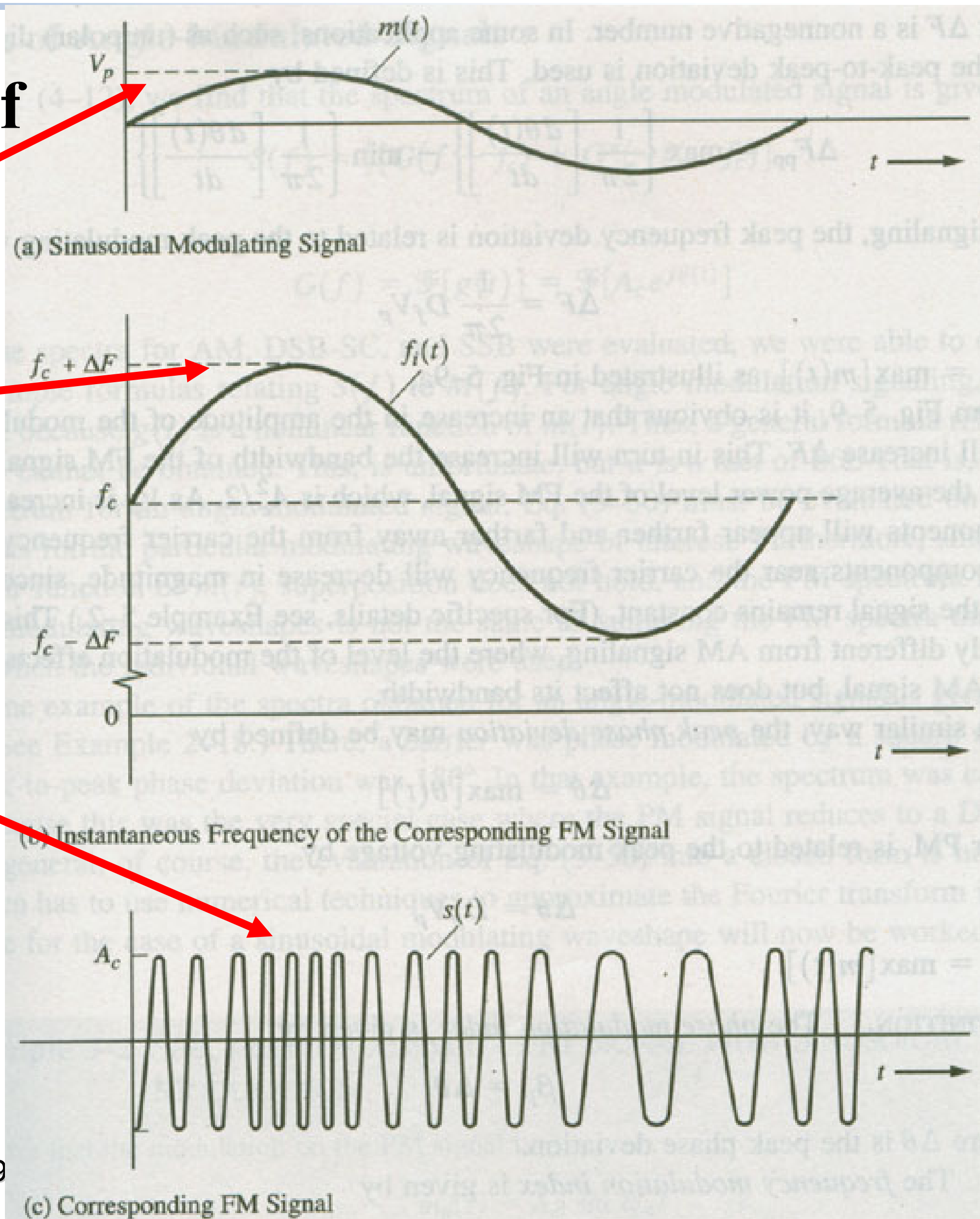
⇓

**increase $\Delta F$**

⇓

**increase the bandwidth of the FM signal**

- **but will not affect the average power level of the FM signal , which is $A_c^2/2$**



# PM & FM

In a similar way, the **peak phase deviation** may be defined by:

$$\Delta\theta = \max\left[\theta(t)\right]$$

*Example:*

**For PM:**

$$\Delta\theta = D_p V_p$$

**where**

$$V_p = \max[m(t)]$$

# PM & FM

*Definition* The **phase modulation index** :

$$\beta_p = \Delta\theta$$

Where $\Delta\theta$ is the peak phase deviation

*Definition* The **frequency modulation index** :

$$\beta_f = \frac{\Delta F}{B}$$

Where $\Delta F$ —— peak frequency deviation

$B$ —— bandwidth of the modulating signal

for the case of sinusoidal modulation, B is the frequency $f_m$ of the sinusoid.

# PM & FM

For the case of PM or FM signaling with sinusoidal modulation, if the PM and FM signals have the same peak frequency deviation, then $\beta_p$ is identical to $\beta_f$.

## Spectrum of angle-modulation signals

$$S(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

**where**

$$G(f) = F[g(t)] = F[A_c e^{j\theta(t)}] \qquad (5\text{-}50)$$

**To evaluate the spectrum for an angle-modulated signal, Eq. (5-50) must be evaluated on the case-by-case basis for the particular modulating waveshape of interest.**

## *Example* 5-2

## Spectrum of a PM or FM signal with sinusoidal modulation

Assume that the modulation on PM signal is :

$$m_p(t) = A_m \sin\omega_m t$$

Then:

$$\theta(t) = D_p m(t) = \beta \sin\omega_m t$$

Where the phase modulation index is $\beta = D_p A_m = max[\theta(t)]$.

The complex envelope is

$$g(t) = A_c e^{j\theta(t)} = A_c e^{j\beta \sin\omega_m t}$$

## Example 5-2 con.

g(t) is periodic with period $T_m=1/f_m$. Its Fourier series can be represented by:

$$g(t)=\sum_{n=-\infty}^{n=\infty} c_n e^{jn\omega_n t} \qquad (5\text{-}55)$$

where $$c_n=\frac{A_c}{T_m}\int_{-T_m/2}^{T_m/2}(e^{j\beta\sin\omega_m t})e^{-jn\omega_m t}dt$$

which reduced to

$$c_n=A_c\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{j(\beta\sin\theta-n\theta)}d\theta\right]=A_cJ_n(\beta)$$

$\theta=\omega_m t$

## Example 5-2 con.

**Taking the Fourier transform of Eq.(5-55)， we get:**

$$G(f)=\sum_{n=-\infty}^{n=\infty} c_n \delta(f-nf_m)$$

**or**

$$G(f)=A_c\sum_{n=-\infty}^{n=\infty} J_n(\beta)\delta(f-nf_m)$$

**Using the result in** $S(f)=\frac{1}{2}[G(f-f_c)+G^*(-f-f_c)]$

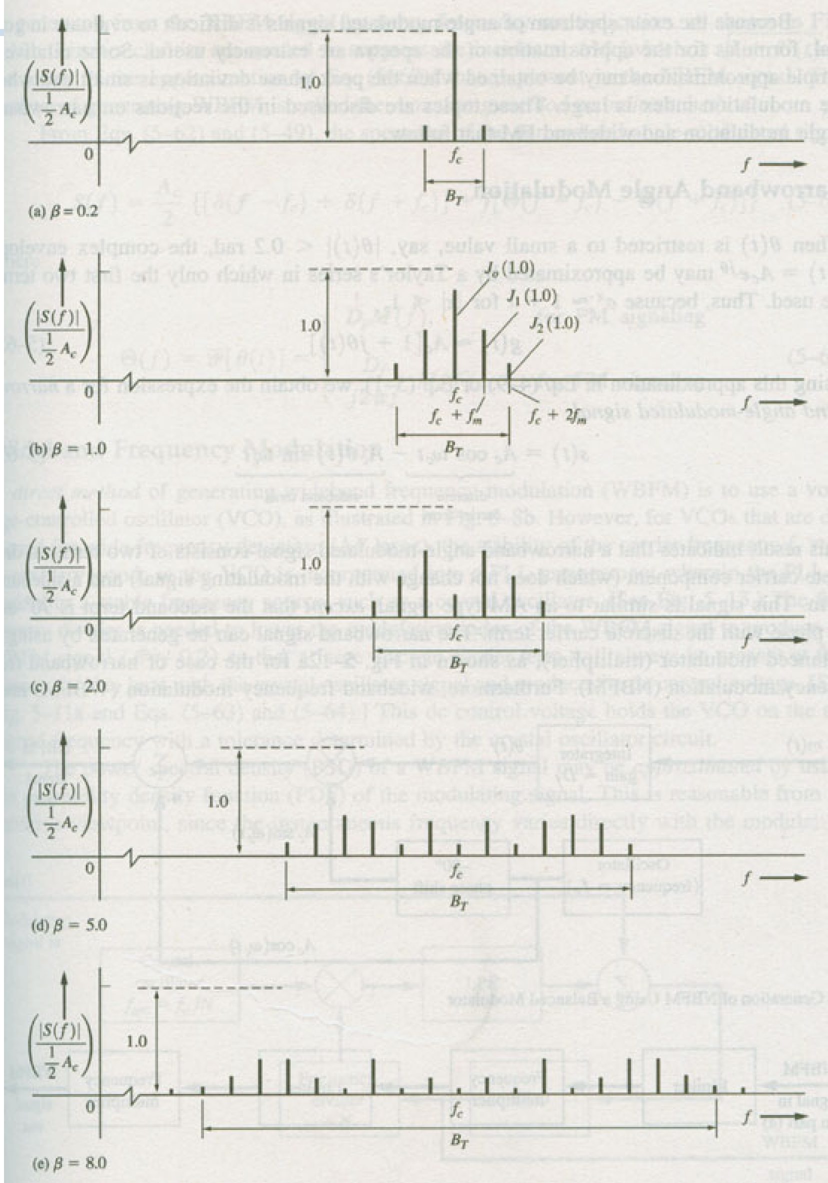
**We may get the spectrum of the angle-modulated signal.**

## Example 5-2 con.

### Carson's rule

**In fact, 98% of the total power is constrained in that bandwidth**

$$B_T = 2(\beta + 1)B$$



# PM & FM

## Narrowband Angle Modulation

When $\theta(t)$ is restricted to a small value, the complex envelope may be approximated by:

$$g(t) = A_c e^{j\theta(t)} \approx A_c[1 + j\theta(t)]$$

- **Then a narrowband angle-modulated signal is:**

$$s(t) = \underbrace{A_c \cos\omega_c t}_{\textit{discrete}\text{ carrier term}} - \underbrace{A_c\theta(t)\sin\omega_c t}_{\textit{sideband}\text{ term}}$$

- **The spectrum of the narrowband angle-modulation signal is:**

$$S(f) = \frac{A_c}{2}\{[\delta(f - f_c) + \delta(f + f_c)] + j[\theta(f - f_c) - \theta(f + f_c)]\}$$

**where** $$\theta(f) = F[\theta(t)] = \begin{cases} D_p M(f) & \text{for PM signaling} \\ \dfrac{D_f}{j2\pi} M(f) & \text{for FM signaling} \end{cases}$$

# Widthband Frequency Modulation

## Generation of WBFM

The key is to solute the stability of the carrier frequency $f_c$.

1. To use a voltage-controlled oscillator (VCO)
2. Phase-locked loops technology

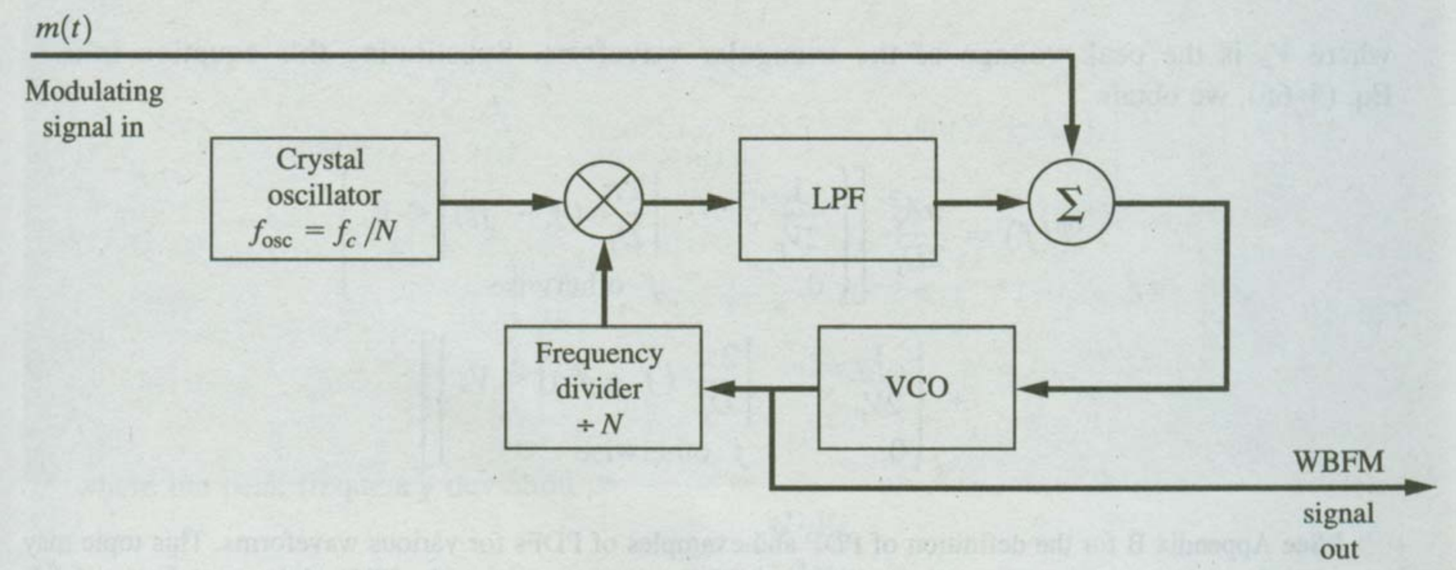



Figure 5–13 Direct method of generating WBFM.

# PM & FM

## Some important properties of angle-modulated signals are:

- The real envelope of an angle-modulated signal is constant, and does not depend on the level of the modulating signal
- An angle-modulated is a nonlinear function of the modulation and the bandwidth of the signal increases as the modulation index increases;
- The discrete carrier level changes depending on the modulating signal;

# 5.7 Frequency-Division multiplexing

# Frequency-Division multiplexing

❖ **Frequency-division multiplexing (FDM) is a technique for transmitting multiple message simultaneously over a wideband channel.**